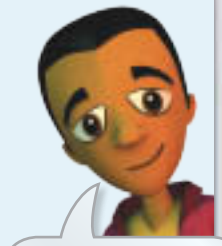
3-5 Parallel Lines and Triangles

Common Core State Standards

G-CO.C.10 Prove theorems about triangles . . . measures of interior angles of a triangle sum to 180°.

MP 1, MP 3, MP 6

Objectives To use parallel lines to prove a theorem about triangles
To find measures of angles of triangles

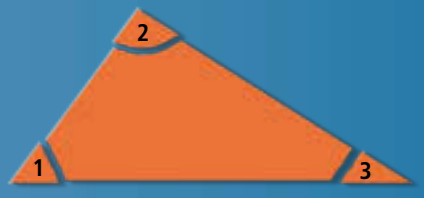


Can you use your result to make a conjecture about other triangles?

MATHEMATICAL PRACTICES

Getting Ready!

Draw and cut out a large triangle. What is the sum of the angle measures of the triangle? Explain. Do not use a protractor. (Hint: Tear off and rearrange the three corners of the triangle.)



In the Solve It, you may have discovered that you can rearrange the corners of the triangle to form a straight angle. You can do this for any triangle.

Essential Understanding The sum of the angle measures of a triangle is always the same.

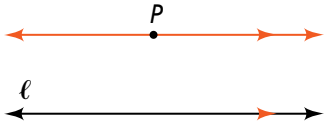
The Solve It suggests an important theorem about triangles. To prove this theorem, you will need to use parallel lines.

Lesson Vocabulary
- auxiliary line
- exterior angle of a polygon
- remote interior angles

take note

Postulate 3-2 Parallel Postulate

Through a point not on a line, there is one and only one line parallel to the given line.

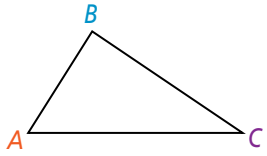


There is exactly one line through P parallel to ℓ.

take note

Theorem 3-11 Triangle Angle-Sum Theorem

The sum of the measures of the angles of a triangle is 180.



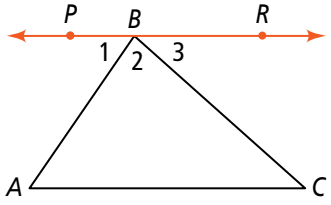
$m\angle A + m\angle B + m\angle C = 180$

The proof of the Triangle Angle-Sum Theorem requires an *auxiliary line*. An **auxiliary line** is a line that you add to a diagram to help explain relationships in proofs. The red line in the diagram below is an auxiliary line.

Proof Proof of Theorem 3-11: Triangle Angle-Sum Theorem

Given: $\triangle ABC$

Prove: $m\angle A + m\angle 2 + m\angle C = 180$

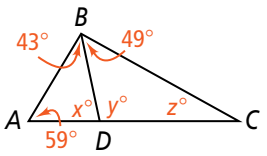


Statements	Reasons
1) Draw $\overleftrightarrow{PR}$ through B, parallel to $\overline{AC}$.	**1)** Parallel Postulate
2) $\angle PBC$ and $\angle 3$ are supplementary.	**2)** $\measuredangle$s that form a linear pair are suppl.
3) $m\angle PBC + m\angle 3 = 180$	**3)** Definition of suppl. $\measuredangle$s
4) $m\angle PBC = m\angle 1 + m\angle 2$	**4)** Angle Addition Postulate
5) $m\angle 1 + m\angle 2 + m\angle 3 = 180$	**5)** Substitution Property
6) $\angle 1 \cong \angle A$ and $\angle 3 \cong \angle C$	**6)** If lines are $\parallel$, then alternate interior $\measuredangle$s are $\cong$.
7) $m\angle 1 = m\angle A$ and $m\angle 3 = m\angle C$	**7)** Congruent $\measuredangle$s have equal measure.
8) $m\angle A + m\angle 2 + m\angle C = 180$	**8)** Substitution Property

When you know the measures of two angles of a triangle, you can use the Triangle Angle-Sum Theorem to find the measure of the third angle.

Problem 1 Using the Triangle Angle-Sum Theorem

Algebra What are the values of x and y in the diagram at the right?



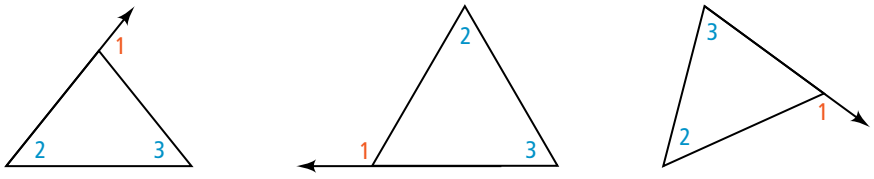
Plan

Which variable should you solve for first?
From the diagram, you know two angle measures in $\triangle ADB$. The third angle is labeled $x°$. So use what you know about the angle measures in a triangle to solve for x first.

Think	Write
Use the Triangle Angle-Sum Theorem to write an equation involving x.	$59 + 43 + x = 180$
Solve for x by simplifying and then subtracting 102 from each side.	$102 + x = 180$ $x = 78$
$\angle ADB$ and $\angle CDB$ form a linear pair, so they are supplementary.	$m\angle ADB + m\angle CDB = 180$
Substitute 78 for $m\angle ADB$ and y for $m\angle CDB$ in the above equation.	$x + y = 180$ $78 + y = 180$
Solve for y by subtracting 78 from each side.	$y = 102$

Got It? **1.** Use the diagram in Problem 1. What is the value of z?

An **exterior angle of a polygon** is an angle formed by a side and an extension of an adjacent side. For each exterior angle of a triangle, the two nonadjacent interior angles are its **remote interior angles.** In each triangle below, $\angle 1$ is an exterior angle and $\angle 2$ and $\angle 3$ are its remote interior angles.

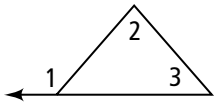


The theorem below states the relationship between an exterior angle and its two remote interior angles.

take note

Theorem 3-12 Triangle Exterior Angle Theorem

The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.



$$m\angle 1 = m\angle 2 + m\angle 3$$

You will prove Theorem 3-12 in Exercise 33.

You can use the Triangle Exterior Angle Theorem to find angle measures.

Problem 2 Using the Triangle Exterior Angle Theorem

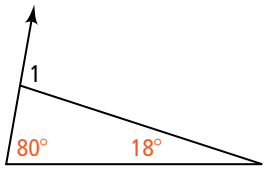
Plan

What information can you get from the diagram?
The diagram shows you which angles are interior or exterior.

A **What is the measure of $\angle 1$?**

$m\angle 1 = 80 + 18$ Triangle Exterior Angle Theorem

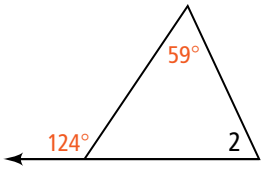
$m\angle 1 = 98$ Simplify.



B **What is the measure of $\angle 2$?**

$124 = 59 + m\angle 2$ Triangle Exterior Angle Theorem

$65 = m\angle 2$ Subtract 59 from each side.



Got It? **2.** Two angles of a triangle measure 53. What is the measure of an exterior angle at each vertex of the triangle?

Problem 3 Applying the Triangle Theorems

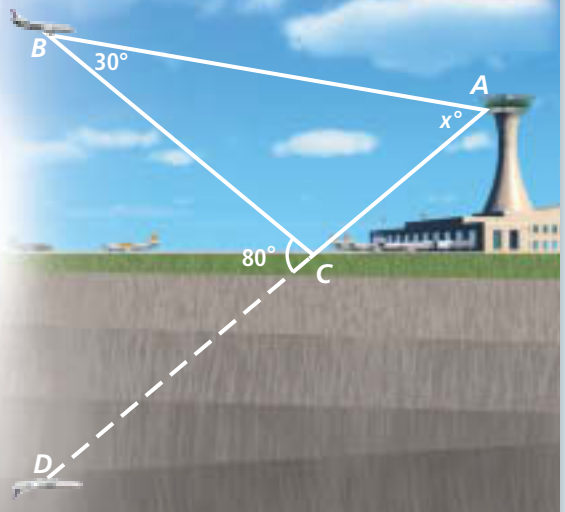
Plan

How can you apply your skills from Problem 2 here?
Look at the diagram. Notice that you have a triangle and information about interior and exterior angles.

Multiple Choice **When radar tracks an object, the reflection of signals off the ground can result in clutter. Clutter causes the receiver to confuse the real object with its reflection, called a ghost. At the right, there is a radar receiver at *A*, an airplane at *B*, and the airplane's ghost at *D*. What is the value of *x*?**

(A) 30 (C) 70

(B) 50 (D) 80



$m\angle A + m\angle B = m\angle BCD$ Triangle Exterior Angle Theorem

$x + 30 = 80$ Substitute.

$x = 50$ Subtract 30 from each side.

The value of x is 50. The correct answer is B.

Got It? **3.** **Reasoning** In Problem 3, can you find $m\angle A$ without using the Triangle Exterior Angle Theorem? Explain.

Lesson Check

Do you know HOW?

Find the measure of the third angle of a triangle given the measures of two angles.

1. 34 and 88
2. 45 and 90
3. 10 and 102
4. x and 50

In a triangle, $\angle 1$ is an exterior angle and $\angle 2$ and $\angle 3$ are its remote interior angles. Find the missing angle measure.

5. $m\angle 2 = 24$ and $m\angle 3 = 106$
6. $m\angle 1 = 70$ and $m\angle 2 = 32$

Do you UNDERSTAND? MATHEMATICAL PRACTICES

7. Explain how the Triangle Exterior Angle Theorem makes sense based on the Triangle Angle-Sum Theorem.

8. **Error Analysis** The measures of the interior angles of a triangle are 30, x, and $3x$. Which of the following methods for solving for x is incorrect? Explain.

A.

$$x + 3x = 30$$
$$4x = 30$$
$$x = 7.5$$

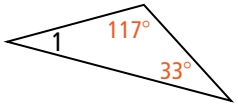
B.

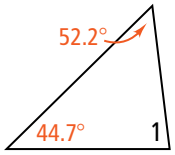
$$x + 3x + 30 = 180$$
$$4x + 30 = 180$$
$$4x = 150$$
$$x = 37.5$$

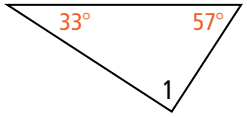
Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

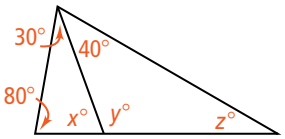
Find $m\angle 1$. See Problem 1.

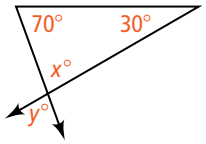
9. 

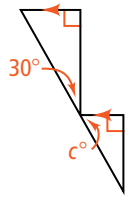
10. 

11. 

Algebra Find the value of each variable.

12. 

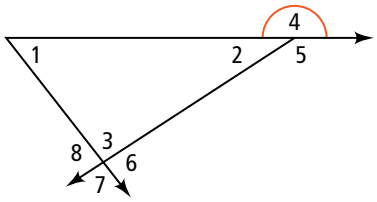
13. 

14. 

Use the diagram at the right for Exercises 15 and 16. See Problem 2.

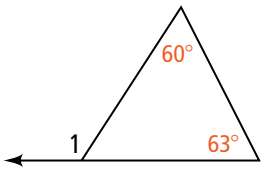
15. **a.** Which of the numbered angles are exterior angles?
    b. Name the remote interior angles for each exterior angle.
    c. How are exterior angles 6 and 8 related?

16. **a.** How many exterior angles are at each vertex of the triangle?
    b. How many exterior angles does a triangle have in all?



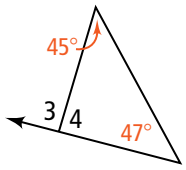
Algebra **Find each missing angle measure.**

17.

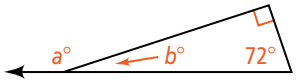


18.

19.



20. A ramp forms the angles shown at the right. What are the values of a and b?



See Problem 3.

21. A lounge chair has different settings that change the angles formed by its parts. Suppose $m\angle 2 = 71$ and $m\angle 3 = 43$. Find $m\angle 1$.

B Apply

Algebra **Use the given information to find the unknown angle measures in the triangle.**

22. The ratio of the angle measures of the acute angles in a right triangle is 1 : 2.

23. The measure of one angle of a triangle is 40. The measures of the other two angles are in a ratio of 3 : 4.

24. The measure of one angle of a triangle is 108. The measures of the other two angles are in a ratio of 1 : 5.

25. Think About a Plan The angle measures of $\triangle RST$ are represented by $2x$, $x + 14$, and $x - 38$. What are the angle measures of $\triangle RST$?

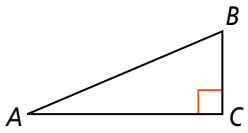
- How can you use the Triangle Angle-Sum Theorem to write an equation?
- How can you check your answer?

26. Prove the following theorem: The acute angles of a right triangle are complementary.

Proof

Given: $\triangle ABC$ with right angle C

Prove: $\angle A$ and $\angle B$ are complementary.

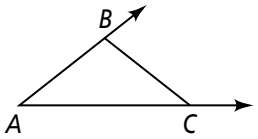


27. Reasoning What is the measure of each angle of an equiangular triangle? Explain.

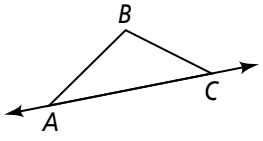
28. Draw a Diagram Which diagram below correctly represents the following description? Explain your reasoning.

Draw any triangle. Label it $\triangle ABC$. Extend two sides of the triangle to form two exterior angles at vertex A.

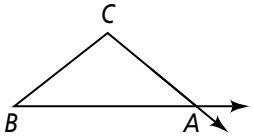
I.



II.

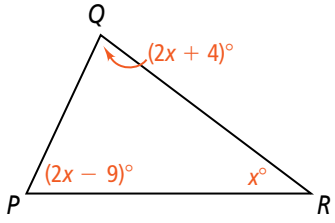


III.



Find the values of the variables and the measures of the angles.

29.



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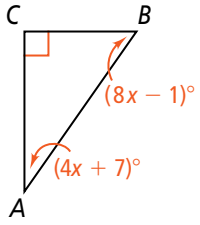

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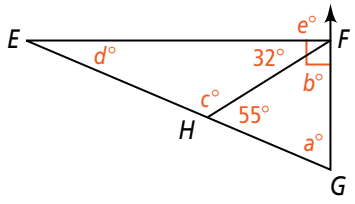

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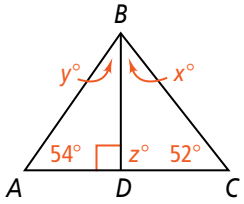
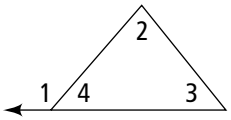


33. Prove the Triangle Exterior Angle Theorem (Theorem 3-12).

Proof

The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.



Given: $\angle 1$ is an exterior angle of the triangle.

Prove: $m\angle 1 = m\angle 2 + m\angle 3$

34. Reasoning Two angles of a triangle measure 64 and 48. What is the measure of the largest exterior angle of the triangle? Explain.

35. Algebra A right triangle has exterior angles at each of its acute angles with measures in the ratio 13 : 14. Find the measures of the two acute angles of the right triangle.

Challenge

Probability **In Exercises 36–40, you know only the given information about the measures of the angles of a triangle. Find the probability that the triangle is equiangular.**

36. Each is a multiple of 30.

37. Each is a multiple of 20.

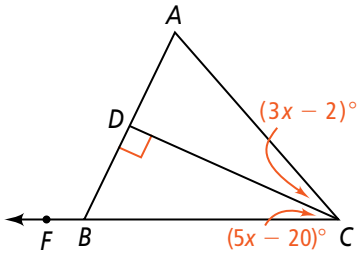
38. Each is a multiple of 60.

39. Each is a multiple of 12.

40. One angle is obtuse.

41. In the figure at the right, $\overline{CD} \perp \overline{AB}$ and $\overline{CD}$ bisects $\angle ACB$. Find $m\angle DBF$.

42. If the remote interior angles of an exterior angle of a triangle are congruent, what can you conclude about the bisector of the exterior angle? Justify your answer.



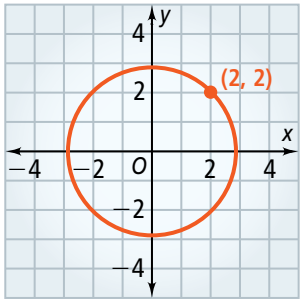
Standardized Test Prep

SAT/ACT

43. The measure of one angle of a triangle is 115. The other two angles are congruent. What is the measure of each of the congruent angles?

Ⓐ 32.5 Ⓑ 57.5 Ⓒ 65 Ⓓ 115

44. The center of the circle at the right is at the origin. What is the approximate length of its diameter?



Ⓕ 2 Ⓗ 5.6

Ⓖ 2.8 Ⓘ 8

45. One statement in a proof is "$\angle 1$ and $\angle 2$ are supplementary angles." The next statement is "$m\angle 1 + m\angle 2 = 180$." Which is the best justification for the second statement based on the first statement?

Ⓐ The sum of the measures of two right angles is 180.

Ⓑ Angles that form a linear pair are supplementary.

Ⓒ Definition of supplementary angles

Ⓓ The measure of a straight angle is 180.

Extended Response

46. $\triangle ABC$ is an obtuse triangle with $m\angle A = 21$ and $\angle C$ is acute.

a. What is $m\angle B + m\angle C$? Explain.

b. What is the range of whole numbers for $m\angle C$? Explain.

c. What is the range of whole numbers for $m\angle B$? Explain.

PERFORMANCE TASK

Apply What You've Learned

MATHEMATICAL PRACTICES
MP 3

Look back at the blueprint on page 139 of the plan for a city park.

a. Choose three triangles in the blueprint. Use the Triangle Angle-Sum Theorem to write an equation relating the measures of the angles of each of your triangles.

b. Choose one of the triangles you looked at in part (a). Find the measure of each angle of your triangle. Show all your steps and explain your reasoning.